Light Fermion Masses in Superstring Derived Standard–like Models

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ABSTRACT

I discuss the suppression of the lightest generation fermion mass terms in realistic superstring standard–like models in the free fermionic formulation. The suppression of the mass terms is a consequence of horizontal symmetries that arise due to the $Z_2 \times Z_2$ orbifold compactification. In a specific model I investigate the possibility of resolving the strong CP puzzle by a highly suppressed up quark mass. In some scenarios the up quark mass may be as small as $10^{-8} MeV$. I show that in the specific model the suppression of the up quark mass is incompatible with the requirement of a nonvanishing electron mass. I discuss how this situation may be remedied.

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Introduction

Electroweak precision data from LEP indicate that the top quark may be found in the mass range $110 - 200 GeV$. On the other hand the up and down quark masses are of the order of $O(1MeV)$, while the well known electron mass is $0.5MeV$. This vast separation of scales is one of the clues to the physics beyond the standard model. In a theory of electroweak symmetry breaking the expected mass of the top quark is rather natural as arising from a renormalizable operator with a Yukawa coupling, λ_t, of order one. On the other hand the mass of the lightest generation states require Yukawa couplings of the order $10^{-5}\lambda_t$. It may also be that $\lambda_u \approx 0$ is consistent with current algebra results and that the mass of the up quark arises from nonperturbative strong interaction effects rather than the value of the high energy parameter, thus providing a solution to the strong CP problem [1]. In this paper I discuss the problem of the suppression of the lightest generation mass terms in the context of realistic superstring derived standard–like models. I discuss the possible solution to the strong CP problem by the suppression of the up quark Yukawa coupling in the context of these models. To suppress CP violation in strong interactions requires $\theta_{\text{tot}}(z/(1+z)) < 10^{-9}$ where $\theta_{\text{tot}} = \theta_{\text{QCD}} + \theta_{\text{quarks}}$ and $z = m_u/m_d$ [2]. I argue that in some scenarios the up quark mass can be as small as $10^{-8}MeV$.

The superstring standard–like models [3,4,5,6] are constructed in the free fermionic formulation [7]. To study the suppression of the lightest generation mass terms I focus on the model that was presented in Ref. [5]. The standard–like models are generated by sets of eight basis vectors, $\{\mathbf{1}, S, b_1, b_2, b_3, \alpha, \beta, \gamma\}$. The set $\{\mathbf{1}, S, b_1, b_2, b_3, 2\gamma\}$ is common to all the realistic models in the free fermionic formulation. The set $\{\mathbf{1}, S, \mathbf{1} + b_1 + b_2 + b_3, 2\gamma\}$ generates a toroidal compactified model with $N = 4$ space–time supersymmetry and $SO(12) \times SO(16) \times SO(16)$ gauge symmetry. The vectors b_1 and b_2 correspond to moding out the six dimensional torus by a $Z_2 \times Z_2$ discrete symmetry with standard embedding, [8,9]. The vectors α, β, γ differ between models and correspond to different choices of Wilson line in the orbifold language. The various choices of vectors α, β, γ and of

the phases $c\begin{pmatrix} \alpha, \beta, \gamma \\ \mathbf{1}, S, b_j \end{pmatrix}$ fix the physical spectrum and determine the low energy effective theory of the superstring standard–like models.

The full massless spectrum together with the quantum numbers were given in Ref. [5]. Here I summarize briefly the states that play a role in the fermion mass matrices. The gauge group after all GSO projections have been applied is $\{SU(3) \times SU(2) \times U(1)_{B-L} \times U(1)_{T_{3_R}} \times U(1)^6\}_O \times \{SU(5)_H \times SU(3)_H \times U(1)^2\}_H$, where the first curly brackets correspond to the observable gauge group that arises from the first $SO(16)$ times $SO(12)$. The second curly brackets arises from the second $SO(16)$. The sectors b_1, b_2 and b_3 correspond to the three twisted sectors of the orbifold model and produce three 16 of $SO(10)$ decomposed under $SU(3) \times SU(2) \times U(1)_{B-L} \times U(1)_{T_{3_R}}$ with charges under the horizontal symmetries.

The Neveu–Schwarz (NS) sector corresponds to the untwisted sector and produces in addition to the gravity and gauge multiplets three pairs of electroweak scalar doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$, three pairs of $SO(10)$ singlets with observable $U(1)$ charges, $\{\Phi_{12}, \Phi_{23}, \Phi_{13}, \bar{\Phi}_{12}, \bar{\Phi}_{23}, \bar{\Phi}_{13}\}$, and three scalars that are singlets of the entire four dimensional gauge group, ξ_1, ξ_2, ξ_3.

The sector $S + b_1 + b_2 + \alpha + \beta$ ($\alpha\beta$ sector) produces in addition to one pair of electroweak doublets, $h_{45}, \bar{h}_{45}$, and one pair of color triplets, seven pairs of $SO(10)$ singlets with horizontal $U(1)$ charges, $\{\Phi_{45}, \bar{\Phi}_{45}, \Phi^{\pm}_{1,2,3}, \bar{\Phi}^{\pm}_{1,2,3}\}$.

In addition to the states from these sectors, which transform solely under the observable gauge group, the neutral states from the sectors $b_j + 2\gamma$ and the sectors $b_{1,2} + b_3 + \alpha + \gamma$ play a role in the fermion mass matrices. The sectors $b_j + 2\gamma$ produce 16 vector representation of the hidden $SO(16)$ gauge group decomposed under $SU(5)_H \times SU(3)_H \times U(1)^2$, $\{T_{1,2,3}, \bar{T}_{1,2,3}, V_{1,2,3}, \bar{V}_{1,2,3}\}$. These states are singlets of the observable $SO(10)$ gauge group but are charged under the horizontal $U(1)^6$ charges. The states from the sectors $b_{1,2} + b_3 + \alpha + \gamma$, $\{H_{13}, H_{14}, H_{17}, H_{18}, H_{19}, H_{20}, H_{23}, H_{24}, H_{25}, H_{26}\}$, are standard model singlets but carry $U(1)_{Z'}$ charge, where $U(1)_{Z'}$ is the $U(1)$ inside $SO(10)$ that is orthogonal to

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the electroweak hypercharge.

The cubic level superpotential and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators, $A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle$, where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators. The nonvanishing terms must be invariant under all the symmetries of the string models and must satisfy all the string selections rules [10]. To obtain the correct ghost charge ($N_b - 1$) of the bosonic vertex operators have to be picture changed from the -1 ghost picture to the 0 ghost picture. The invariance under the global left–moving $U(1)$ symmetries and the Ising model correlators must be checked after all picture changing operations have been performed. The invariant terms are extracted by using a simple FORTRAN code. In Ref. [11] I discussed the properties of the standard–like models which simplify the analysis of nonrenormalizable terms.

The cubic level superpotential is given by,

$$
\begin{aligned}
W = &\{(u_{L_1}^c Q_1 \bar{h}_1 + N_{L_1}^c L_1 \bar{h}_1 + u_{L_2}^c Q_2 \bar{h}_2 + N_{L_2}^c L_2 \bar{h}_2 + u_{L_3}^c Q_3 \bar{h}_3 + N_{L_3}^c L_3 \bar{h}_3) + h_1 \bar{h}_2 \bar{\Phi}_{12} + h_1 \bar{h}_3 \bar{\Phi}_{13} \\
&+ h_2 \bar{h}_3 \bar{\Phi}_{23} + \bar{h}_1 h_2 \Phi_{12} + \bar{h}_1 h_3 \Phi_{13} + \bar{h}_2 h_3 \Phi_{23} + \Phi_{23} \bar{\Phi}_{13} \Phi_{12} + \bar{\Phi}_{23} \Phi_{13} \bar{\Phi}_{12} + \bar{\Phi}_{12}(\bar{\Phi}_1^+ \bar{\Phi}_1^- \\
&+ \bar{\Phi}_2^+ \bar{\Phi}_2^- + \bar{\Phi}_3^+ \bar{\Phi}_3^-) + \Phi_{12}(\Phi_1^- \Phi_1^+ + \Phi_2^- \Phi_2^+ + \Phi_3^- \Phi_3^+) + \frac{1}{2}\xi_3(\Phi_{45}\bar{\Phi}_{45} + h_{45}\bar{h}_{45} + D_{45}\bar{D}_{45} \\
&+ \Phi_1^+ \bar{\Phi}_1^+ + \Phi_1^- \bar{\Phi}_1^- + \Phi_2^+ \bar{\Phi}_2^+ + \Phi_2^- \bar{\Phi}_2^- + \Phi_3^+ \bar{\Phi}_3^+ + \Phi_3^- \bar{\Phi}_3^-) + h_3 \bar{h}_{45} \Phi_{45} + \bar{h}_3 h_{45} \bar{\Phi}_{45}\} \\
&+ \{\frac{1}{2}[\xi_1(H_{19}H_{20} + H_{21}H_{22} + H_{23}H_{24} + H_{25}H_{26}) + \xi_2(H_{13}H_{14} + H_{15}H_{16} + H_{17}H_{18})] \\
&+ \bar{\Phi}_{23}H_{24}H_{25} + \Phi_{23}H_{23}H_{26} + h_2 H_{16}H_{17} + \bar{h}_2 H_{15}H_{18} + e_{L_1}^c H_{10}H_{27} + e_{L_2}^c H_8 H_{29} + (V_1 H_9 \\
&+ V_2 H_{11})H_{27} + V_6 H_5 H_{29} + \bar{\Phi}_{45}H_{17}H_{24} + D_{45}H_{18}H_{21} + h_{45}H_{16}H_{25}\} \qquad (1)
\end{aligned}
$$

where a common normalization constant $\sqrt{2}g$ is assumed. From Eq. (2) it is seen that only $+\frac{2}{3}$ charged quarks obtain a cubic level mass term. This result arises due to the assignment of boundary conditions in the vector γ [6]. Mass terms for $-\frac{1}{3}$ and for charged leptons must be obtained from nonrenormalizable terms. The light Higgs spectrum is determined by the massless eigenstates of the doublet Higgs mass matrix. The doublet mass matrix consists of the terms $h_i \bar{h}_j \langle \Phi^n \rangle$, and is defined by

$h_i(M_h)_{ij}\bar{h}_j$, $i, j = 1, 2, 3, 4$ where $h_i = (h_1, h_2, h_3, h_{45})$ and $\bar{h}_i = (\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_{45})$. At the cubic level of the superpotential the Higgs doublets mass matrix is given by,

$$
M_h = \begin{pmatrix}
0 & \bar{\Phi}_{12} & \bar{\Phi}_{13} & 0 \\
\Phi_{12} & 0 & \bar{\Phi}_{23} & 0 \\
\Phi_{13} & \Phi_{23} & 0 & \Phi_{45} \\
0 & 0 & \bar{\Phi}_{45} & 0
\end{pmatrix}.
\tag{2}
$$

The superstring standard–like models contain an "anomalous" $U(1)$ gauge symmetry. The "anomalous" $U(1)$ symmetry generates a Fayet–Iliopolous D–term at the one loop level that breaks supersymmetry at the Planck scale and destabilize the vacuum [12]. Supersymmetry is restored by giving a VEV to some standard model singlets in the spectrum along F and D flat directions. In the standard–like models, it has been found that we must impose [5,4,11],

$$
\langle \Phi_{12}, \bar{\Phi}_{12}, \xi_3 \rangle = 0,
\tag{3}
$$

and that Φ_{45}, and $\bar{\Phi}_{13}$ or $\bar{\Phi}_{23}$, must be different from zero. From this result it follows that in any flat F and D solution, h_3 and $\bar{h}_3$ obtain a Planck scale mass. This result is a consequence of the symmetry of the vectors α and β with respect to the b_1 and b_2 sectors [11]. The implication is that h_3 and $\bar{h}_3$ do not contribute to the light Higgs representations. Consequently, the mass terms for the states from the sector b_3 will be suppressed.

At the cubic level of the superpotential there are two pairs of light Higgs doublets which may consist of combinations of $\{h_1, h_2, h_{45}\}$ and $\{\bar{h}_1, \bar{h}_2, \bar{h}_{45}\}$. At the nonrenormalizable level of the superpotential, additional non vanishing entries in the Higgs mass matrix appear [11,8], rendering one additional pair supermassive. The light Higgs representations typically consist of $\bar{h}_1$ or $\bar{h}_2$ and h_{45}, depending on the additional nonvanishing terms in the Higgs mass matrix [11]. In the analysis of nonrenormalizable terms I search for any terms that include (h_1, h_2, h_{45}) and $(\bar{h}_1, \bar{h}_2, \bar{h}_{45})$ and thus do not make an assumption as to what are the specific light

Higgs combinations. The suppression of the light fermion masses will be shown to be independent of this choice. However, to examine whether one can obtain models in which the strong CP problem is resolved by a sufficiently suppressed up quark mass, I will make the assumption that the light Higgs representations are $\bar{h}_1$ and h_{45}.

Among the realistic models in the free fermionic formulation, the standard–like models have the unique property that there are three and only three chiral generations. Therefore, the identification of the three light generations is unambiguous.

2. Light fermion mass terms

In Ref. [8] it was shown that the global left–moving horizontal symmetry $U(1)_{\ell_3}$ forbid the formation of terms of the form $f_3 f_3 h \phi^n$ or $f_3 f_3 \bar{h} \phi^n$, where f_3 are fermions from the sectors b_3, h and $\bar{h}$ are combinations of $\{h_1, h_2, h_{45}\}$ and $\{\bar{h}_1, \bar{h}_2, \bar{h}_{45}\}$ respectively, and ϕ^n is a combination of $SO(10)$ singlet fields from the Neveu–Schwarz sector and the sector $b_1 + b_2 + \alpha + \beta$. In this paper I extend the analysis to the case where ϕ^n include scalar fields from the sectors $b_j + 2\gamma$ and $b_{1,2} + b_3 + \alpha + \beta$.

At the quintic level the following mass terms are obtained

$$d_2 Q_2 h_{45} \bar{\Phi}_2^- \xi_1, \ \ e_2 L_2 h_{45} \bar{\Phi}_2^+ \xi_1, \ \ d_1 Q_1 h_{45} \Phi_1^+ \xi_2, \ \ e_1 L_1 h_{45} \Phi_1^- \xi_2 \tag{4a}$$

$$u_2 Q_2 (\bar{h}_{45} \Phi_{45} \bar{\Phi}_{23} + \bar{h}_1 \bar{\Phi}_i^+ \Phi_i^-), \ \ u_1 Q_1 (\bar{h}_{45} \Phi_{45} \bar{\Phi}_{13} + \bar{h}_2 \Phi_i^+ \Phi_i^-) \tag{4b}$$

$$(u_2 Q_2 h_2 + u_1 Q_1 h_1) \frac{\partial W}{\partial \xi_3}. \tag{4c}$$

At order $N = 6$ we obtain mixing terms for $-\frac{1}{3}$ charged quarks,

$$d_3 Q_2 h_{45} \Phi_{45} V_3 \bar{V}_2, \ \ d_2 Q_3 h_{45} \Phi_{45} V_2 \bar{V}_3, \ \ d_3 Q_1 h_{45} \Phi_{45} V_3 \bar{V}_1, \ \ d_1 Q_3 h_{45} \Phi_{45} V_1 \bar{V}_3, \tag{5}$$

and for charged leptons

$$e_3 L_2 h_{45} \Phi_{45} T_3 \bar{T}_2, \ \ e_2 L_3 h_{45} \Phi_{45} T_2 \bar{T}_3, \ \ e_3 L_1 h_{45} \Phi_{45} T_3 \bar{T}_1, \ \ e_1 L_3 h_{45} \Phi_{45} T_1 \bar{T}_3, \tag{6}$$

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At order $N = 7$ we obtain in the down quark sector,

$$d_2 Q_1 h_{45} \Phi_{45} (V_1 \bar{V}_2 + V_2 \bar{V}_1) \xi_i, \qquad d_1 Q_2 h_{45} \Phi_{45} (V_1 \bar{V}_2 + V_2 \bar{V}_1) \xi_i, \qquad (7a, b)$$

$$d_1 Q_3 h_{45} \Phi_{45} V_3 \bar{V}_1 \xi_2, \qquad d_3 Q_1 h_{45} \Phi_{45} V_1 \bar{V}_3 \xi_2, \qquad (7c, d)$$

$$d_2 Q_3 h_{45} \Phi_{45} V_3 \bar{V}_2 \xi_1, \qquad d_3 Q_2 h_{45} \Phi_{45} V_2 \bar{V}_3 \xi_1, \qquad (7e, f)$$

where $\xi_i = \{\xi_1, \xi_2\}$. In the up quark sector we obtain,

$$u_1 Q_2 \bar{h}_1 \Phi_{45} \{\bar{\Phi}_2^- (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \bar{\Phi}_1^+ (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} \qquad (8a)$$

$$u_2 Q_1 \bar{h}_1 \Phi_{45} \{\bar{\Phi}_1^- (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \bar{\Phi}_2^+ (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} \qquad (8b)$$

$$u_1 Q_2 \bar{h}_2 \Phi_{45} \{\Phi_2^+ (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \Phi_1^- (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} \qquad (8c)$$

$$u_2 Q_1 \bar{h}_2 \Phi_{45} \{\Phi_1^+ (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \Phi_2^- (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} \qquad (8d)$$

$$u_3 Q_1 \bar{h}_1 \Phi_{45} \{\bar{\Phi}_1^- T_1 \bar{T}_3 + \bar{\Phi}_3^+ V_3 \bar{V}_1\} \quad u_1 Q_3 \bar{h}_1 \Phi_{45} \{\bar{\Phi}_3^- T_1 \bar{T}_3 + \bar{\Phi}_1^+ V_3 \bar{V}_1)\} \qquad (8e)$$

$$u_3 Q_1 \bar{h}_2 \Phi_{45} \{\Phi_3^+ T_1 \bar{T}_3 + \Phi_1^- V_3 \bar{V}_1\} \quad u_1 Q_3 \bar{h}_2 \Phi_{45} \{\Phi_3^+ T_1 \bar{T}_3 + \Phi_1^- V_3 \bar{V}_1)\} \qquad (8f)$$

$$u_3 Q_2 \bar{h}_1 \Phi_{45} \{\bar{\Phi}_2^- T_2 \bar{T}_3 + \bar{\Phi}_3^+ V_3 \bar{V}_2\} \quad u_2 Q_3 \bar{h}_1 \Phi_{45} \{\bar{\Phi}_3^- T_2 \bar{T}_3 + \bar{\Phi}_2^+ V_3 \bar{V}_2)\} \qquad (8g)$$

$$u_3 Q_2 \bar{h}_2 \Phi_{45} \{\Phi_2^+ T_2 \bar{T}_3 + \Phi_3^- V_3 \bar{V}_2)\} \quad u_2 Q_3 \bar{h}_2 \Phi_{45} \{\Phi_3^- T_2 \bar{T}_3 + \Phi_2^- V_3 \bar{V}_2)\} \qquad (8h)$$

From the terms in Eqs. (4–8) we can construct fermion mass matrices that lead to quark mass and mixing spectrum of the correct order of magnitude [13]. However, diagonal mass terms for the states from the sector b_3 do not appear in the equations above. Potential diagonal mass terms for the lightest generation states are of the form $Q_3 d_3 h \phi^n$ and $Q_3 u_3 \bar{h} \phi^n$, where h and $\bar{h}$ are combinations of $\{h_1, h_2, h_{45}\}$ and $\{\bar{h}_1, \bar{h}_2, \bar{h}_{45}\}$ respectively, and ϕ^n is a string of standard model singlets. The standard model singlets divide into several classes: (i) $SO(10)$ singlets from the Neveu–Schwarz sector and the sector $S + b_1 + b_2 + \alpha + \beta$. (ii) $SO(10)$ singlets from the sectors $b_j + 2\gamma$. (iii) States that carry $U(1)_{Z'}$ charges from the sectors $b_{1,2} + b_3 + \alpha + \beta$.

Invariance under the left–moving global $U(1)_{\ell_3}$ symmetry forbids the formation of terms $f_3 f_3 h \phi^n$ and $f_3 f_3 \bar{h} \phi^n$, where ϕ^n are restricted to class (i) singlets. The

argument goes as follows. The fermions from the sector b_3 carry $U(1)_{\ell_3} = \frac{1}{2}$. The bosons from the Neveu–Schwarz sector $\{h_3, \bar{h}_3, \Phi_{12}, \bar{\Phi}_{12}, \xi_3\}$ carry $U(1)_{\ell_3} = -1$, and all the other states from the NS and $\alpha\beta$ sectors have $U(1)_{\ell_3} = 0$. The charges of the twisted and untwisted states under the symmetries $U(1)_{\ell_{1,2,3}}$ are due to the underlying $Z_2 \times Z_2$ orbifold compactification. Since h_3 and $\bar{h}_3$ are supermassive, to form a potential mass term that is invariant under $U(1)_{\ell_3}$, we must tag to $f_3 f_3$, a Higgs state that is neutral under $U(1)_{\ell_3}$ and one or more of $\{\Phi_{12}, \bar{\Phi}_{12}, \xi_3\}$. However, the $U(1)_{\ell_3}$ charges of $\{\Phi_{12}, \bar{\Phi}_{12}, \xi_3\}$ are changed to zero by picture changing and therefore we cannot form a term that is invariant under $U(1)_{\ell_3}$ with only class (i) singlets. Thus, we have to examine terms that include class (ii) and (iii) singlets. Below I focus on a scenario with $\bar{h}_1$ as the light Higgs that couples to $+\frac{2}{3}$ charged quarks. A search up to order $N = 9$ shows that terms that include only class (i) and (ii) singlets do not appear up to order $N = 9$. At order $N = 9$ we obtain for example in the up quark sector,

$$Q_3 u_3 \bar{h}_1 \Phi_{45} \Phi_{45} \{ V_3 \bar{V}_2 \bar{T}_3 T_2 + V_2 \bar{V}_3 \bar{T}_2 T_3 \} \tag{9a}$$

$$Q_3 u_3 \bar{h}_1 \Phi_{45} \xi_1 \{ (\bar{\Phi}_2^- \Phi_{23} + \Phi_2^+ \Phi_{13}) \bar{T}_2 T_2 + (\bar{\Phi}_2^+ \Phi_{23} + \Phi_2^- \Phi_{13}) \bar{V}_2 V_2 \} \tag{9f}$$

with additional terms of the form of Eq.(9) with $\bar{h}_1$ replaced by $\bar{h}_2$ and $\bar{h}_{45}$ to make a total of 35 terms.

If we include terms that break $U(1)_{Z'}$ then we obtain at the quintic order,

$$Q_3 u_3 (\bar{h}_2 H_{24} H_{25} + \bar{h}_{45} H_{17} H_{24}) \tag{10}$$

and at order $N = 6$

$$Q_3 d_3 \{ (h_1 \Phi_3^+ + h_2 \bar{\Phi}_3^-) H_{17} H_{24} + h_{45} \bar{\Phi}_3^- H_{24} H_{25} \} \tag{11}$$

at order $N = 7$ we obtain in the up quark sector,

$$Q_3 u_3 \{ \bar{h}_1 \bar{\Phi}_i^+ \bar{\Phi}_i^- + \bar{h}_2 (\Phi_i^{\pm} \Phi_i^{\pm} + \xi_2 \xi_2 + \Phi_{13} \bar{\Phi}_{13}) \} H_{24} H_{25} \tag{12a}$$

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$$Q_3 u_3 \bar{h}_2 \Phi_{45} \xi_2 H_{18} H_{25} \tag{12b}$$

$$Q_3 u_3 \bar{h}_{45} \Phi_{45} \left(\frac{\partial W_3}{\partial \xi_1} + \frac{\partial W_3}{\partial \xi_2}\right) \xi_i \xi_j \tag{12c}$$

Plus additional terms of the form $Q_3 u_3 (\bar{h}_2 + \bar{h}_{45}) H^4$ where $\langle H \rangle$ breaks $U(1)_{Z'}$.

The texture of the quark mass matrices is determined by the choice of singlet VEVs. The singlet VEVs are constrained by the F and D flat constraints. In a general solution we may obtain quark masses of order MeV. For example assuming assuming $\langle H \rangle \approx 0$ and taking the F and D flat direction from Ref. [13] the quark mass matrices take the form

$$M_u \sim \begin{pmatrix} \epsilon & \frac{V_3 \bar{V}_2 \Phi_{45} \bar{\Phi}_3^+}{M^4} & 0 \\ \frac{V_3 \bar{V}_2 \Phi_{45} \bar{\Phi}_2^+}{M^4} & \frac{\bar{\Phi}_i^- \bar{\Phi}_i^+}{M^2} & \frac{V_1 \bar{V}_2 \Phi_{45} \bar{\Phi}_2^+}{M^4} \\ 0 & \frac{V_1 \bar{V}_2 \Phi_{45} \bar{\Phi}_1^+}{M^4} & 1 \end{pmatrix} v_1, \tag{13}$$

and

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_3 \bar{V}_2 \Phi_{45}}{M^3} & 0 \\ \frac{V_3 \bar{V}_2 \Phi_{45} \xi_1}{M^4} & \frac{\bar{\Phi}_2^- \xi_1}{M^2} & \frac{V_1 \bar{V}_2 \Phi_{45} \xi_i}{M^4} \\ 0 & \frac{V_1 \bar{V}_2 \Phi_{45} \xi_i}{M^4} & \frac{\Phi_1^+ \xi_2}{M^2} \end{pmatrix} v_2, \tag{14}$$

where $v_1 = \langle \bar{h}_1 \rangle$, $v_2 = \langle h_{45} \rangle$ and $\epsilon \sim 0$. The up, down quark masses and the Cabibbo angle are given by

$$m_u \approx \frac{(M_u)_{12}(M_u)_{21}}{(M_u)_{22}} = \frac{\langle V_3 \bar{V}_2 \Phi_{45} \rangle^2 \langle \bar{\Phi}_3^+ \bar{\Phi}_2^+ \rangle}{M^6 \langle \bar{\Phi}_i^+ \bar{\Phi}_i^- \rangle} v_1 \tag{15}$$

and

$$m_d \approx \frac{(M_d)_{12}(M_d)_{21}}{(M_d)_{22}} = \frac{\langle V_3 \bar{V}_2 \Phi_{45} \rangle^2 \langle \xi_1 \rangle}{M^6 \langle \bar{\Phi}_2^- \rangle \langle \xi_1 \rangle} v_2 \tag{16a}$$

$$\sin \theta_c \approx \frac{(M_d)_{12}}{(M_d)_{22}} = \frac{\langle V_3 \bar{V}_2 \Phi_{45} \rangle}{M \langle \bar{\Phi}_2^- \rangle \langle \xi_1 \rangle} \tag{16b}$$

To make an attempt at a numerical estimate of the up and down quark masses I take the F and D flat solution that was found in Ref. [13],

$$\frac{1}{3} |\frac{\langle \bar{V}_2 \rangle}{M}|^2 = \frac{1}{2} |\frac{\langle V_3 \rangle}{M}|^2 = \frac{1}{6} |\frac{\langle \Phi_{45} \rangle}{M}|^2 = |\frac{\langle \bar{\Phi}_3^+ \rangle}{M}|^2 = 2 |\frac{\langle \bar{\Phi}_2^+ \rangle}{M}|^2 = \frac{g^4}{16 \pi^2} \tag{17}$$

and $\frac{\langle \bar{\Phi}_i^+ \bar{\Phi}_i^- \rangle}{M^2} \approx 0.01$. With this solution and with $\langle \xi_1 \rangle \sim \frac{1}{4}M$ we obtain Cabibbo angle and down quark mass of the correct order of magnitude [13]. From Eq. (15) and (17), we observe that Taking $g \sim 0.8$ the unification scale [15], and $v_1 \sim 100 GeV$ we obtain naively $m_u \sim 0.01 MeV$ from this solution. In a general solution we may assume $\langle \phi \rangle / M \sim 0.1$, which yields $m_u \sim 0.1 MeV$.

The interesting observation regarding the quark mass matrices is that for particular choices of flat directions the textures of the down and up mass matrices is different [13]. This entails the possibility that the diagonal entry $(M_u)_{11}$ one of the non–diagonal entries in the up quark mass matrix $(M_u)_{12}$ or $(M_u)_{21}$ vanish up to some order [13]. Consequently, it is possible that the mass of the lightest up quark state vanishes or is highly suppressed, while the down quark and the Cabibbo angle are of the correct order of magnitude. This is an interesting possibility as it may provide a solution to the strong CP problem.

First, I estimate the possible contribution from the order $N = 9$ terms Eqs. (9). I impose that only one state from a given sector $b_j + 2\gamma$ gets a VEV, $\{V_1, \bar{V}_2, V_3\}$. Therefore, the only terms that contribute are those that contain the condensates of the hidden $SU(5)$ gauge group. The bilinear hidden sector condensates produce a suppression factor that is given by

$$(\frac{\Lambda_5}{M})^2 = \exp(\frac{2\pi}{b} \frac{(1 - \alpha_0)}{\alpha_0}) \tag{18}$$

where $b = \frac{1}{2}n_5 - 15$. For $n_5 = 6$ and $\alpha_0 = (1/20 - 1/25)$, $\Lambda_5 \sim (10^{12} - 10^{14}) GeV$. Taking, $\Lambda_5 \sim 10^{14} GeV$ and $\phi/M \approx (1/10 - 1/25)$ I estimate the contribution of the $N = 9$ order term to be in the range $m_u \sim (10^{-6} - 10^{-8}) MeV$, where I took $v_1 = \langle \bar{h}_1 \rangle \sim 100 GeV$ and multiplied by a factor of ten to account for the multiplicity of terms. If we take $\Lambda_5 \sim 10^{13} GeV$ then we obtain $m_u \sim (10^{-9} - 10^{-11}) MeV$.

If in addition we assume that $\Lambda_{Z'}$ is suppressed, say $\Lambda_{Z'} \leq 10^{10} GeV$, then the contribution from the terms that break $U(1)_{Z'}$ is suppressed by at least $(\Lambda_{Z'}/M)^2 \sim 10^{-16}$. Thus, these terms are sufficiently suppressed and produce $m_u \sim 10^{-11} MeV$.

In Ref. [11] it was argued that VEVs that break $U(1)_{Z'}$ have to be suppressed because of the constraint that higher order nonrenormalizable terms should not spoil the cubic level F and D flat solution. The constraints on the $U(1)_{Z'}$ breaking VEVs are investigated further in Ref. [16], where it is suggested that already at the cubic level the F flatness constraints restrict the $U(1)_{Z'}$ breaking VEVs. However, lacking an understanding of the SUSY breaking mechanism, the $U(1)_{Z'}$ VEVs may still be large, say of the order $10^{14} GeV$, and produce soft SUSY breaking terms that are in accord with the naturalness constraints on the SUSY spectrum. In this case the up quark mass will be of the order $10^{-3} MeV$, not small enough to resolve the strong CP puzzle. Thus, to obtain m_u sufficiently small we have to assume $\Lambda_{Z'} \leq 10^{11} GeV$.

In Eq. (9) we obtained nonvanishing terms of the form $u_3 Q_3 \bar{h}_1 \phi^n$ where ϕ are class (i) and (ii) singlets only. These terms are suppressed by $(\Lambda_5/M)^2 \sim (10^{-8} - 10^{-10})$. However, there may exist higher order terms ($N > 9$) with only class (i) and (ii) singlets that are not suppressed by the hidden sector condensation scale. In this case the singlets ϕ belong to the set $\langle\phi\rangle \in \{\{NS\}, \{\alpha\beta\}, V_1, \bar{V}_2, V_3\}$. At order $N = 11$ we obtain the terms,

$$Q_3 u_3 \bar{h}_1 (\Phi_{45} V_3 \bar{V}_2)^2 (\Phi_2^- \bar{\Phi}_3^+ + \Phi_3^- \bar{\Phi}_2^+). \tag{19}$$

Inserting the numerical values from the solution in Eq. (17) and with $(\langle\Phi_3^-\rangle/M) = (3g^2)/(\sqrt{104}4\pi)$ [13], we obtain $\lambda_u \sim 10^{-9}$ or $m_u \approx 10^{-4} MeV$. Thus, to construct model in which m_u is sufficiently suppressed we have to impose $\langle\Phi_3^-\rangle \approx 0$. Imposing $\langle\Phi_3^-\rangle = 0$ and $\langle\bar{\Phi}_3^+\rangle = 0$ then guarantees that the diagonal mass terms are suppressed up to order $N = 11$. At order $N = 12$ there are no invariant terms of the desired form. At order $N = 13$ we obtain

$$Q_3 u_3 \bar{h}_1 (\Phi_{45} V_1 \bar{V}_2)^2 (\Phi_{23} \bar{\Phi}_2^+ \bar{\Phi}_1^+ + \Phi_{13} \bar{\Phi}_1^+ \Phi_2^- + \Phi_{13} \bar{\Phi}_2^+ \Phi_1^-)(\xi_1 + \xi_2) \tag{20}$$

Thus, in the best case scenario the diagonal mass terms will be suppressed up to order $N = 13$. Inserting the numerical values from the solution in Ref. [13] and with $g \sim (0.8 - 0.7)$ we obtain $\lambda_u \sim (10^{-12} - 12^{-13})$ or $m_u \sim (10^{-7} - 10^{-8}) MeV$.

Next I examine the contribution from the nondiagonal terms. If we choose a flat F and D direction solution with $\langle \bar{\Phi}_3^+ \rangle = 0$ then $(M_u)_{12}$ in Eq. (13) vanishes up to order $N = 7$, while from Eq. (4a) we observe that to give mass to the μ lepton we must have $\langle \bar{\Phi}_2^+ \rangle \neq 0$. Thus, to examine whether it is possible to obtain m_u sufficiently suppressed to resolve the strong CP problem I focus on the terms that contribute to $(M_u)_{12}$. Eq. (8g) shows that at order $N = 7$ there is contribution from condensates of the hidden $SU(5)$ gauge group. This contribution is estimated to be $(M_u)_{12} < 10^{-10} v_1$, where I have taken $\Lambda_5 \sim 10^{14} GeV$ and $\langle \phi \rangle \sim 1/10$. From Eq. (15) this contributes less than $O(10^{-7} MeV)$ to m_u. Next, I examine terms that contribute to $(M_u)_{12}$ which are not suppressed by hidden sector condensates. These terms must be of the form $u_3 Q_2 \bar{h}_1 V_3 \bar{V}_2 \phi^n$, where $n = 1, \cdots, N - 5$, and $\langle \phi \rangle \in \{\{NS\}, \{\alpha\beta\}, V_1, \bar{V}_2, V_3\}$. At order $N = 8$ there are no potential terms contributing to $u_3 Q_2 \bar{h}_1 V_3 \bar{V}_2$. At order $N = 9$ we obtain the following potential terms

$$u_3 Q_2 \bar{h}_1 \Phi_{45} \Phi_{45} V_3 \bar{V}_2 \bar{\Phi}_3^+ (\xi_i \xi_i + \xi_1 \xi_2 + \Phi_{13} \bar{\Phi}_{13} + \Phi_{23} \bar{\Phi}_{23} + \frac{\partial W_3}{\partial \xi_3}) \qquad (21a)$$

$$u_3 Q_2 \bar{h}_1 \Phi_{45} \Phi_{45} V_3 \bar{V}_2 \Phi_3^- (\frac{\partial W_3}{\partial \Phi_{12}}) \qquad (21b)$$

Plus additional terms that are suppressed by $(\Lambda_{Z'}/M)^2$. The requirement $\bar{\Phi}_3^+ = 0$ imposes that the terms in Eq.(21a) vanish identically. The terms in Eq. (21b) are suppressed by the cubic level F flatness constraint $(\partial W_3/\partial \Phi_{12}) = 0$. Thus, the order $N = 9$ terms are suppressed by at least $10^{-4}(M_{\text{SUSY}}/M_{Pl})^2 \leq 10^{-12}$. At order $N = 10$ there are no terms that contribute to $(M_u)_{12}$ which are not suppressed by hidden sector condensates. At order $N = 11$ and higher there will be many additional terms. Invariance under $U(1)_{r_6}$ necessitates that either $\bar{\Phi}_3^+$ or Φ_3^- appear in the correlators $u_3 Q_2 \bar{h}_1 V_3 \bar{V}_2 \phi^n$. Thus, all these terms vanish if we impose $\langle \bar{\Phi}_3^+ \rangle \approx 0$ and $\langle \Phi_3^- \rangle \approx 0$ on the F and D flat solution.

In summary, the diagonal mass terms for the lightest generation states are suppressed due to the horizontal symmetry $U(1)_{\ell_3}$. The suppression of the lightest generation states results from the basic structure of the vectors α and β with

respect to the sectors b_1, b_2, b_3 and the resulting constraints on flat directions [8]. Therefore, the suppression of the lightest generation states is expected to be a general characteristic of the standard–like models. In fact, in the flipped $SU(5)$ superstring model, where vectors similar to α and β are constructed, similar constraints on flat directions are obtained [14]. The up and down quark mass matrices possess a different texture. This enables obtaining Cabibbo angle and down quark mass of the correct order of magnitude while suppressing the up quark mass. With the assumption that $\bar{h}_1$ and h_{45} are the light Higgs representations, to obtain an up quark mass that is highly suppressed the following constraints must be imposed: (1) Only one state from each sector $b_j + 2\gamma$ can obtain a VEV. (2) VEVs which break $U(1)_{Z'}$ have to be suppressed with $\Lambda_{Z'} < 10^{11} GeV$. (3) $SU(5)$ condensation scale of order $O(10^{13} GeV)$. (4) $\langle \bar{\Phi}_3^+ \rangle \approx 0$ and $\langle \Phi_3^- \rangle \approx 0$. In this case the leading contribution to $(M_u)_{12}$ is from terms that are suppressed by hidden sector condensates and the contribution to the diagonal mass term is from order $N = 13$ terms which are not suppressed by hidden sector condensates. In this best case scenario the up quark mass can be as small as $10^{-8} MeV$.

Next I address the problem of the electron mass. The electron mass is obtained from diagonal terms, $e_3 L_3 h_{45} \langle \phi \rangle^n$, and nondiagonal terms, $e_{2,3} L_{3,2} h_{45} \langle \phi \rangle^n$. Eq. (6) shows that order $N = 6$ there are terms that are suppressed by the hidden sector condensation scale $(\Lambda_5 / M)^2$. Assuming a solution with $(M_e)_{22} \sim 10^{-3}$ and taking $\langle h_{45} \rangle \approx 100 GeV$ produces $m_e \sim 10^{-8} MeV$. Clearly too small. At order $N = 6$ an additional term that breaks $U(1)_{Z'}$ is obtained,

$$e_3 L_3 h_{45} \bar{\Phi}_3^+ H_{24} H_{25}. \tag{22}$$

Assuming, $\Lambda_{Z'} \sim 10^{14} GeV$, $\langle \phi \rangle \sim 1/10$ and $\langle h_{45} \rangle \sim 100 GeV$ we get $m_e \sim 10^{-4} MeV$. At order $N = 8$ we get

$$e_2 L_3 h_{45} \Phi_{45} \Phi_2^- \bar{\Phi}_3^+ V_3 \bar{V}_2 \qquad\qquad e_3 L_2 h_{45} \Phi_{45} \Phi_3^- \bar{\Phi}_2^+ V_3 \bar{V}_2 \tag{23a, b}$$

Taking $\langle \phi \rangle \sim M/10$ yields $m_e \sim 10^{-2} MeV$. Up to order $N = 12$ all diagonal terms, $e_3 L_3$, are suppressed by at least $(\Lambda_5 / M)^2$ or $(\Lambda_{Z'} / M)^2$. At order $N =$

12 we obtain terms that are suppressed only by singlet VEVs of order $M/10$. However, invariance under $U(1)_{r_6}$ dictates that to all order of nonrenormalizable terms Φ_3^- and $\bar{\Phi}_3^+$ must appear in the correlators of the form $e_3 L_3 h_{45} \phi^n$, where $\phi \in \{\{NS\}, \{\alpha\beta\}, V_1, \bar{V}_2, V_3\}$.

From the discussion above it is evident that in this model the suppression of m_u and an electron mass of order $(0.1 MeV)$ are incompatible as the first requires $\langle \bar{\Phi}_3^+ \rangle \approx 0$ and $\langle \Phi_3^- \rangle \approx 0$, while the second requires the opposite. Replacing $\bar{h}_1$ by $\bar{h}_2$ will produce similar results as it does not affect the invariance of the correlators under $U(1)_{r_6}$. A remedy to this situation may be obtained by modifying the phases $c \begin{pmatrix} b_j \\ \alpha, \beta, \gamma \end{pmatrix}$. The left– right–moving horizontal symmetries $U(1)_{\ell, r_{4,5,6}}$ fix the invariant nonrenormalizable terms. The horizontal symmetries and the Ising model operators arise from the internal fermionic states $\{y, \omega | \bar{y} \bar{\omega}\}$ which correspond to the six dimensional compactified space in an orbifold formulation [8,9]. Modifying the phases $c \begin{pmatrix} b_j \\ \alpha, \beta, \gamma \end{pmatrix}$ modifies the GSO projections and consequently the charges under these horizontal symmetries. Thus for example it may be possible to choose phases that will produce electron nondiagonal mass terms at order $N = 7$ that are suppressed only by singlet VEVs of order $M/10$, while the corresponding up quark mass terms will be pushed to higher orders. A similar dependence of nonrenormalizable terms on the boundary conditions of the fermionic states $\{y, \omega | \bar{y}, \bar{\omega}\}$ was found in the case of quartic and quintic order bottom quark mass terms [4,6]. Thus, this is a viable possibility and merits further investigation.

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